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Semantic Realism: A New Philosophy for Quantum Physics

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The epistemological position underlying the standard interpretation of quantum physics (QP) can be classified as a form of *verificationism* : to be precise, *empirical* verificationism (nontestable physical statements have no meaning). This position can be criticized and maintained to be the deep root of many quantum paradoxes. Semantic Realism proposes an alternative viewpoint, according to which even nontestable statements made up of individually testable statements have a meaning, but quantum laws are not necessarily true in physical contexts that QP itself classifies as nonaccessible. This viewpoint produces a new interpretation of QP which preserves its formal structure and observational interpretation, but invalidates those theorems that aim to prove such puzzling features of this theory as *nonlocality* and *contextuality* (Bell and Bell–Kochen–Specker theorems).

1. INTRODUCTION

It is a basic notion in quantum physics (QP) that the properties of physical systems are *nonobjective*. As Mermin (1993) writes,

NO. "It is a fundamental quantum doctrine that a measurement does not, in general, reveal a preexisting value of the measured property."

This doctrine challenges not only our physical imagination, but even the basic procedures of our reasoning within natural languages. Indeed, it is a primary function of any language to attribute properties to things and deduce, via general laws, further properties. If NO is accepted, this function must be completely reconsidered. Thus, one could wonder whether it is worth maintaining NO, as the standard interpretation of QP does. But it is well known that the connected theorems of *Bell* and *Bell–Kochen–Specker* (briefly *Bell-KS*) seem to prove *nonlocality* and *contextuality* of QP, respectively, and

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that NO follows directly from contextuality. Thus, NO appears to ensue from a technical result that is independent of one's philosophical point of view, which implies that one could not renounce it without rejecting QP. Hence most people accept NO even when proposing new interpretations or modifications of QP.

However, the above convictions can be questioned. Indeed, any physical theory contains an interpretation of the mathematical formalism which strongly depends on the epistemological choices underlying the theory. Hence, the way in which a mathematical result is interpreted may change when these choices are changed. In the case of QP, the deduction of contextuality, hence of NO, from the Bell-KS theorem rests on an interpretation that follows from the standard philosophy of QP itself. But this philosophy contains, in particular, NO, so that the claim that this follows from the Bell-KS theorem independently of any philosophical choice is incorrect. Therefore one can imagine that some changes in the standard philosophy might produce a new interpretation of QP which does not imply, on one hand, a modification of its formal apparatus, thus saving the core of the quantum description of the world, and which, on the other hand, allows one to falsify or read in a nonstandard way the Bell and Bell-KS results, thus avoiding nonlocality and contextuality and reconciling QP with the basic procedures of our ordinary reasoning. I will discuss here the main ideas underlying *Semantic Realism* (SR), which is a recent attempt to provide such an interpretation of QP (Garola, 1991; Garola and Solombrino, 1996a, b).

2. CRITICIZING EMPIRICAL VERIFICATIONISM

In order to realize the program illustrated in the Introduction, the first step necessarily consists of a critical analysis of the standard philosophy of QP. Therefore, let me recall that the general attitude underlying NO is to regard as "metaphysical" every attempt at introducing physical entities which, in principle, cannot be observed: consequently, *no statement which cannot be verified by means of a suitable measurement procedure has a truth value and can be accepted as meaningful in the language of physics.*

The above position will be called *empirical verificationism* here, since it collapses the concept of truth into the concept of empirical verification, both for elementary and for complex statements (a statement is *elementary* if it attributes a property, or, equivalently, a value of some observable, to an individual sample of a physical system; it is *complex* if it is obtained by modifying or connecting elementary statements by means of connectives such as "not," "and," "or," etc., and/or by means of quantifiers such as "for every" and "exists"). At first glance empirical verificationism may seem to express only a physicist's natural refusal of statements that cannot be justified on experimental grounds. But a deeper insight shows that it has a number of profound consequences. Let me discuss some of these.

(i) Eliminating all statements of a physical language that are meaningless according to empirical verificationism may affect the deduction rules, which then turn out to depend on the physical theory that one wants to express by means of the language itself. This entails that one cannot state any *a priori* rationality criterion which is independent of the theory. The collapse of truth and empirical access to truth produces the collapse of logic into physics (see, e.g., Putnam, 1969; Finkelstein, 1969, 1972). In the case of QP a number of paradoxes appear, which follow from applying physical rules to logical arguments.

(ii) The elementary statements of the language of QP cannot be divided once and for all into meaningful and meaningless. Indeed, there are physical contexts in which some statements are meaningful and others are meaningless, and different contexts in which the opposite occurs. This is an obvious consequence of the Heisenberg uncertainty principle. But in this way subjectivity enters into physics, at least in the sense that, by choosing the physical context, one chooses which properties are meaningful and which are meaningless for the object being studied.

(iii) Consider a complex statement in the language of QP that is obtained by connecting elementary statements attributing properties to a given sample of a physical system by means of the connectives "and," "or," etc. Empirical verificationism says that this statement is meaningful if and only if it can be verified. This means that an apparatus must exist which yields one of the outcomes *true* and *false* whenever applied to the sample. But, then, this apparatus can be looked at as testing a physical property. Hence, our complex statement is logically equivalent to an elementary statement that attributes this property to the sample. This implies that the former statement can be meaningful if and only if it is logically equivalent to an elementary statement, which constitutes a particular but relevant aspect of the collapse of the logical structure into an empirical structure discussed in (i).

The above consequences of the basic philosophy of QP have deeply influenced physicists' conception of the world in our century. But one can wonder whether empirical verificationism is unquestionable and whether it is so inherent to QP that it could not be renounced without losing all physical knowledge provided by QP itself. The answer to both these questions is negative in my opinion. Indeed, it is well known that a number of logicians and epistemologists argued against the verificationist concept of truth, observing that the concept of verification presupposes that something is verified, which is just the truth of the statement that one is considering: thus, verification and truth cannot be identified. This objection applies in particular to empirical verificationism; one can then add that the standard justification for adopting this position, that is, freeing physics from old metaphysical hindrances, confuses the *semantic* concept of truth (which can be defined by means of a set-theoretic model, as in classical logic) with the *ontological* concept, according to which asserting that a statement is true means assuming the actual existence of the entities that are mentioned in it. Finally, it has been seen in the Introduction that NO, hence the philosophical choices underlying empirical verificationism, are not unavoidable, and that some alternative positions can be conceived. One of these (SR) will be schematically presented in the next sections.

3. THE ESSENTIALS OF SEMANTIC REALISM

The main aim of SR is to provide a general logical and linguistic framework suitable for expressing a wide set of physical theories (among them QP) which avoids the paradoxes following from the collapse of logic into physics induced by empirical verificationism. Thus, the basic choice of SR is the rejection of this position. But this choice does not specify the logic that one has to adopt when constructing a language for physics. However, the criticism in Section 2 suggests avoiding all logics resting on a verificationist concept of truth, even if not empirical (in particular, intuitionistic logic). On the other hand, the convenience of making quantum reasoning closer to ordinary reasoning strongly recommends the adoption of classical logic. Thus, this adoption is explicitly done by SR.

The natural subsequent step consists in constructing a language for physical theories that has classical logic built in. This could be done informally, as usual in physics: but this procedure would not prevent all semantic ambiguities inherent in the use of a natural language. Alternatively, one could provide a completely formalized language: but this would be exceedingly complicated. Therefore, SR adopts a compromise, accepting the standard language of physics as its general language and formalizing mainly that part of it which is interpreted on the observative domain (the *observative language*).

Let me provide an insight into the basic ideas inspiring this formalization.

(i) One introduces the set I of *laboratories*, i.e., space-time domains in the actual world.

(ii) One introduces the sets Π and $\mathcal{R}$ of *preparations* and *dichotomic registering devices*, respectively. These notions come out from considering the simplest experiment (*yes–no experiment*) that can be conceived. Indeed, this consists of a device (the preparation) that prepares an individual sample of a given physical system (briefly, a *physical object*) and of a second device (the dichotomic registering device) that performs a test on the sample, yielding one of two possible outcomes (yes/no).

(iii) In every physical theory there are preparations, or registering devices, that are considered physically equivalent. Hence, preparations can be grouped into equivalence classes, called *states* (the set of all states is denoted by $\mathcal{S}$ here). Analogously, dichotomic registering devices can be grouped into equivalence classes, called *effects* (the set of all effects is denoted by $\mathcal{F}$ here).

(iv) In every laboratory $i \in I$ one can consider all physical objects that are prepared by repeating the same preparation at different times or by activating different preparations. The set of all these objects is called the *domain* D_i of i. Inside D_i, one can select every object that has been prepared by a preparation belonging to a given state S: the subset of all these objects is called the *extension* $\rho_i(S)$ of S in i. Analogously, one can select inside D_i every object that *would* give the answer yes if a test should be performed on it by means of a dichotomic registering device belonging to a given effect F: the subset of all these objects is called the *extension* $\rho_i(F)$ of F in i.

It must be noted that the definition of extension of an effect F is not acceptable according to the standard quantum philosophy. Indeed, consider another effect G made up by dichotomic registering devices that are noncompatible with those in F and verify which elements are in $\rho_i(G)$ by means of one of these devices: then, the extension of F cannot be defined, since it has no meaning to refer to what "would have happened" if one had verified which elements are in $\rho_i(F)$ by means of a device belonging to F. This conclusion, however, follows from accepting NO. The contrary assumption that the extensions of effects are defined subtends the rejection of NO, and stands on the fact that the extension of *any* effect F can be actually exhibited in a laboratory i if no other effect is considered. But, of course, the extensions of different effects cannot generally be exhibited conjointly.

(v) The above definitions allow one to construct a formalized language L which has the sets $\mathcal{S}$ and $\mathcal{F}$ as sets of predicates and is endowed with a built-in classical logical structure. Indeed, one assumes that an elementary statement of the form $S(x)$ (*the physical object x is in the state S*) or $F(x)$ (*the physical object would induce answer yes in the effect F*) is true in the laboratory i if and only if the object x belongs to the extension in i of S or F, respectively. The truth in i of complex statements obtained by using elementary statements, connectives, and quantifiers is then defined by means of standard conventions in classical logic.

The definition of a truth value for every statement in L contrasts with empirical verificationism and has deep consequences. Indeed, the interpretation of the predicates in L shows that L satisfies the operational requirement that the truth value of every elementary statement can be tested, that is, all elementary statements of L are individually *testable*. But, if one considers a complex statement of L, it may occur that the elementary statements that appear in it contain effects that are noncompatible according to the physical theory that one wants to express by means of L: in this case, the truth values of these statements cannot be tested conjointly, and the complex statement has a truth value, but it is *nontestable* . Thus, the fundamental epistemological distinction between *truth* and *testability* is recovered in L. This distinction entails that nontestable statements of L may play a role whenever inferential procedures are carried out in L, which reconciles the "logic" of L with ordinary reasoning and eliminates a number of paradoxes following from modifying logical rules in the name of physics.

4. STATES, EFFECTS, AND QUANTUM LOGIC

In the sets of states and effects a number of further definitions and properties can be introduced. Let me collect here the most important of them.

(i) The set of all extensions of states is a *partition* of D_i. This is a typical feature of SR which rests on the idea that the same physical object cannot be thought of as prepared by two physically inequivalent preparations. On the contrary, the extensions of two (or more) different effects may have nonempty intersection. Therefore, one can introduce the following *partial order relation* $\subset$ on the set of all effects:

Let F, G be effects; then $F \subset G$ iff, for every $i \in I$, $\rho_i(F) \subseteq \rho_i(G)$.

Thus, one recovers in an SR context the poset of all effects, which is a familiar structure in the current literature on the foundations of QP.

(ii) By exchanging the yes/no outcomes in all devices belonging to a given effect F, one obtains the *complement* F^c of F, which is such that, for every $i \in I$, $\rho_i(F^c) = D_i \setminus \rho_i(F)$.

(iii) One convenes that preparations and dichotomic registering devices are chosen in such a way that one can define a *probability of the yes outcome* that is the limit (in the statistical sense) to which frequencies approach in every laboratory whenever the number of elements in any ensemble that is considered becomes large.

(iv) For every state S one introduces the *certainly true domain* $\mathscr{F}_S$ of S, defined by setting $\mathscr{F}_S = \{F \in \mathscr{F} | \text{for every } i \in I, \rho_i(S) \subseteq \rho_i(F)\}$. For every $i \in I$ one then considers the intersection $\hat{\rho}_i(S) = \cap_{F \in \mathscr{F}_S} \rho_i(F)$. This set is obviously such that $\rho_i(S) \subseteq \hat{\rho}_i(S)$ and plays a crucial role in SR. Indeed, $\hat{\rho}_i(S)$ can be used in order to introduce the set $\mathscr{S}_P$ of *pure* states: a state S is said to be pure if and only if no state $S' \neq S$ exists such that, in every laboratory i, $\rho_i(S') \subseteq \hat{\rho}_i(S)$. Furthermore, $\hat{\rho}_i(S)$ can be used in order to introduce a *preclusivity* relation $\perp$ on the set of pure states: for every S, $S' \in \mathscr{S}_P$, $S \perp S'$ iff, in every laboratory i, $\rho_i(S') \cap \hat{\rho}_i(S) = \emptyset$.

(v) By introducing a notion of *closure* for every subset of pure states via the preclusivity relation, one can select within $\mathscr{F}$ the subposet $\mathscr{F}_e$ of all *exact effects*, or *dichotomic observables*. Every $F \in \mathcal{F}_e$ is associated to a *testable* property, and every dichotomic registering device in F can be used for testing whether the physical property associated to F holds for a given physical object x. One can then argue that the correspondence between the set of exact effects and the set of testable properties is one-to-one. Hence, the two sets are identified (of course, testable properties are represented by projections in standard QP).

The introduction of the concept of testable property requires an idealization, since only *ideal* dichotomic registering devices (which can at most be "approached" by actual devices) can be used in order to test exactly whether a given property holds or not for a given physical object. More important, a testable property can only be attributed to a(n) (individual) physical object and tested on it by means of a single act of testing: in logical terms, it is a *first-order property*. Therefore, a testable property must not be confused with physical properties that refer to *ensembles* of physical objects [like the frequencies considered in (iii)], or even to ensembles of ensembles (for instance, when one says that a given frequency is minimal in a given state): indeed in logical terms these are second- and third-order properties, respectively. Even these higher order properties can be tested: but their test consists of a (usually great) number of elementary tests of testable properties and of a comparison of the sets of results that have been obtained (*correlation measurements*).

(vi) In the SR approach two different binary relations of compatibility can be introduced on the set $\mathcal{F}_e$ of all testable properties of L_e, as follows.

Semantic compatibility: The testable properties F_1, F_2 are semantically compatible iff they can be simultaneously true for a physical object x.

Pragmatic compatibility (or *conjoint testability*): The testable properties F_1, F_2 are pragmatically compatible (conjointly testable) iff they can be simultaneously measured on a physical object x (hence this relation can be identified in QP with the standard relation of compatibility).

(vii) Let $S \in \mathcal{S}_P$, $i \in I$, and come back to $\hat{\rho}_i(S)$. There is no *a priori* reason for maintaining that an effect exists which, whatever i may be, has just $\hat{\rho}_i(S)$ as extension. Whenever this occurs, this effect is minimal in $\mathcal{F}_S$ according to the order $\subset$ defined in (i), is called the *testable support* of S, and is denoted by F_S.

The existence of a testable support F_S for every pure state S can be easily deduced in QP if one refers to the standard interpretation of this theory (if $|\varphi\rangle$ is a normalized vector representing the state S, F_S is the testable property represented by the projection $|\varphi\rangle\langle\varphi|$). Yet, this existence can be seriously questioned whenever *compound* physical systems are considered. Indeed if S is an *entangled* state, it can be argued that one cannot find in QP any dichotomic registering device characterizing an effect which is minimal in $\mathscr{F}_S$. Hence, no effect can be considered to be the support of S. This entails that not all projections necessarily represent (first-order) testable properties in QP. Because of this objection, SR does not assume that pure states necessarily have a testable support, and defines entangled states (which may exist or not in a given theory) as the pure states that have no testable support. In the case of QP this definition constitutes a new branching point in which the interpretation provided by SR differs from the standard interpretation.

(viii) The above characterization of entangled states entails that the poset of all testable properties is not necessarily endowed with a lattice structure within SR. But one can resort to *completion* procedures that adjoin further elements to the poset of all testable properties and transform it into a lattice (Garola, 1985). The new elements have no operational interpretation, but they can be considered as *theoretical* (first-order) properties. Thus, one obtains an enlarged set $\mathscr{E}_e$ of properties which is partitioned into a set $\mathscr{F}_e$ of testable properties and a set $\mathscr{D}_e$ of theoretical properties (even entangled states then have a support, but this is a theoretical, not a testable property).

(ix) The completion $(\mathscr{E}_e, \subset)$ of the poset $(\mathscr{F}_e, \subset)$ of exact effects allows one to recover a structure of (complete, orthocomplemented) lattice, which can be identified, in the case of QP, with the lattices that appear in a number of axiomatic approaches to this theory (e.g., Mackey, 1963; Jauch, 1968; Piron, 1976) and that are represented by the lattice of all projections in standard QP. Thus "quantum logic" is recovered by SR as a physical, not a logical structure which, however, contains in the case of compound systems even elements that are nonobservative (the theoretical properties).

5. MEANING, TESTABILITY, AND QUANTUM "PARADOXES"

The set $\mathscr{E}_e$ of testable and theoretical properties can be used for constructing a new language L_e. This has $\mathscr{S}$ and $\mathscr{E}_e$ (in place of $\mathscr{S}$ and $\mathscr{F}$) as sets of predicates, is endowed with the same (classical) logical structure of L, and contains, as usual, elementary and complex statements. Furthermore, even the statements in L_e can be divided into *testable* and *nontestable* . The former have a truth value that can be empirically checked, the latter have a conventional truth value, as it occurs in L: but in the case of L_e there are also elementary statements that are nontestable, since $\mathscr{E}_e$ contains theoretical properties.

All statements of L_e can be used in order to state physical laws. But one must then distinguish between *empirical laws*, which are expressed by testable statements, and *theoretical* laws, which are expressed by nontestable statements. Then SR maintains that, according to the operational spirit of QP, theoretical laws must be considered formal structures whose role consists in producing, via logical deduction and *auxiliary assumptions*, or *premises*, empirical physical laws that can be directly tested. The premises usually consist in stating some testable properties for physical objects which define a *physical context*, that is, a particular physical situation in which the general laws are to be applied. Now, different kinds of physical contexts occur, which can be classified by using the compatibility relations introduced in Section 4. To be precise, a context is *contradictory* (or *impossible*), *nonaccessible*, or *accessible* if semantically noncompatible, semantically but not pragmatically compatible, or semantically and pragmatically compatible properties, respectively, are assumed. Then the operational viewpoint suggests that statements expressing empirical laws can be maintained to be true only in accessible physical contexts (they could be false in nonaccessible contexts) since these are the only contexts in which the theory itself allows one to check the validity of the laws. SR explicitly accepts this suggestion by introducing a new epistemological principle (*metatheoretical generalized principle*, or, briefly, MGP) which limits the domain of validity of physical laws, renouncing the extrapolation of our knowledge beyond its empirical limits. This principle can be stated informally as follows.

MGP. A statement expressing an empirical physical law (deduced or not from a general theoretical law) is true in every physical context in which only semantically and pragmatically compatible properties are assumed for each physical object that is considered (accessible context).

By using MGP, it has been proved (e.g., Garola and Solombrino, 1996b) that the existing proofs of the Bell and Bell-KS theorems rest on assuming the validity of some empirical correlation laws deduced from the general theoretical laws of QP outside the domain of validity established by MGP. This entails that nonlocality and contextuality do not hold within the SR interpretation of QP, which confirms, in particular, the self-consistency of this interpretation. Let me discuss briefly the general lines of the SR criticism of the aforesaid theorems.

Consider first the Bell theorem (Bell, 1964). Its more ancient proofs essentially show that locality implies inequalities which are not consistent with QP. In all these proofs, empirical physical laws are assumed as true in nonaccessible physical contexts, which implies a violation of MGP. This is apparent, for instance, in the Wigner (1970) and Sakurai (1985) proofs, where many subsets of physical objects are considered, in each set the spin components along different directions being assigned, and then an inequality is obtained by applying a perfect correlation law to these objects; it is less apparent in other proofs, as in Bell's original one, where an inequality is obtained regarding sets of physical objects in nonaccessible contexts, but the deduction does not introduce quantum laws (I reserve the name *Bell inequality* in the following to Bell-type inequalities obtained in this way): however, these laws are used whenever the Bell inequality is compared with an analogous inequality predicted by QP. Thus, one concludes that none of these proofs can be accepted according to SR. More recent proofs do not resort to inequalities (e.g., Greenberger *et al.*, 1990). Here, a number of empirical laws are deduced from a general theoretical quantum law, and it is implicitly assumed that they all are simultaneously valid. But whenever one of these empirical laws is assumed to hold, the properties predicted by it are pragmatically noncompatible with the properties that one introduces further as premises in order to deduce predictions from another law of the set, so that the validity of the latter law cannot be assured, because of MGP, in the nonaccessible physical context that has thus been defined. One concludes that even the more recent proofs cannot be accepted according to SR.

The invalidation of the above proofs implies that QP does not necessarily conflict with locality. One may then wonder what would happen if one performs a suitable test of a given Bell inequality. Would the inequality be violated or not? The answer of SR is that a Bell inequality is a correct theoretical formula which is not epistemically accessible in QP. Any physical experiment tests something else (correlations among properties of physical objects in accessible contexts) and yields the results predicted by QP. No contradiction can occur, since the inequalities that can be tested in QP could be identified with Bell inequalities only by violating MGP. Thus, a Bell inequality does not provide a method for testing experimentally whether either QP or locality is correct, contrary to a widespread belief. But the fact that quantum inequalities are different from Bell inequalitites proves that something must go wrong with quantum laws regarding compound systems within nonaccessible contexts.

Finally, consider the Bell-KS theorem. This theorem is usually maintained to assert the contextuality of QP, which intuitively means that the truth value of a statement attributing a property to a physical object and belonging to a set of properties that are measured on it depends on the whole set, not only on the state of the object (it is apparent that NO follows at once from contextuality). The original proofs of this property of QP (Bell, 1966; Kochen and Specker, 1967) were rather complicated, but there are some recent proofs (Mermin, 1993) that are quite simple and immediate. By considering these proofs, it has been shown (Garola and Solombrino, 1996b) that they hold under conditions which are invalidated, according to SR, because of the same arguments used in the case of the Greenberger *et al.* proof of nonlocality. To be precise, MGP imposes constraints on the values of physical observables which are weaker than the conditions explicitly stated by Kochen and Specker (1967) or Mermin (1993) whenever these conditions are used repeatedly in order to prove the contextuality of QP.

6. CONCLUDING REMARKS

I would like to conclude by discussing briefly two topics; first, classifying the SR interpretation of QP within the set of all possible interpretations of this theory; second, pointing out some suggestions provided by the SR approach for a theory going beyond QP without contradicting it.

With reference to the first topic, let me recall that a *statistical interpretation* of QP can be opposed to a *realistic interpretation*; within the latter, a further opposition occurs between *completeness/nonobjectivity* interpretations, which assume the completeness of QP together with the doctrine of nonobjectivity of physical properties, and *objectivity/incompleteness* interpretations, which accept the incompleteness of QP together with the doctrine of objectivity of physical properties (Busch *et al.*, 1991). Now, the SR approach gives up empirical verificationism in favor of a theory of truth as correspondence when accepting classical logic, and QP turns out to be incomplete within SR (Garola, 1992). Hence the SR interpretation of QP actually is an objectivity/incompleteness interpretation, provided that the term "objectivity" is endowed with a purely semantic and nonontological meaning.

With reference to the second topic, the following remarks can be made, based on the arguments discussed in Section 4.

(i) The interpretation of the supports of entangled states as theoretical properties shows that the quantum treatment of compound systems by means of tensor products is semantically ambiguous. Indeed, a one-dimensional projection should correspond to a (first-order) testable property in the case of a first-kind state, to a (first-order) theoretical property in the case of a second-kind state. This suggests that a new theory should represent testable and theoretical properties by means of different mathematical entities, so that one can distinguish the former from the latter.

(ii) The invalidation of the proofs of contextuality and nonlocality suggests that, contrary to a widespread belief, noncontextual and local hidden-variables models for QP may exist, but they should propose new laws that differ from QP laws within nonaccessible physical contexts.

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